Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced Level

Friday 24 June 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics FP3), the paper reference (6669), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P35414A

1. The curve C has equation $y = 2x^3$, $0 \le x \le 2$.

The curve C is rotated through 2π radians about the x-axis.

Using calculus, find the area of the surface generated, giving your answer to 3 significant figures.
(5)

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2. (*a*) Given that $y = x \arcsin x$, $0 \le x \le 1$, find

(i) an expression for $\frac{dy}{dx}$,

(ii) the exact value of $\frac{dy}{dx}$ when $x = \frac{1}{2}$.
(3)

(*b*) Given that $y = \arctan(3e^{2x})$, show that

$$\frac{dy}{dx} = \frac{3}{5\cosh 2x + 4\sinh 2x}.$$
(5)

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3. Show that

(*a*) $\int_5^8 \frac{1}{x^2 - 10x + 34}\, dx = k\pi$, giving the value of the fraction k,
(5)

(*b*) $\int_5^8 \frac{1}{\sqrt{(x^2 - 10x + 34)}}\, dx = \ln(A + \sqrt{n})$, giving the values of the integers A and n.
(4)

4.
$$I_n = \int_1^e x^2 (\ln x)^n \, dx, \quad n \ge 0.$$

(*a*) Prove that, for $n \ge 1$,

$$I_n = \frac{e^3}{3} - \frac{n}{3} I_{n-1}.$$
(4)

(*b*) Find the exact value of I_3.
(4)

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5. The curve C_1 has equation $y = 3\sinh 2x$, and the curve C_2 has equation $y = 13 - 3e^{2x}$.

(*a*) Sketch the graph of the curves C_1 and C_2 on one set of axes, giving the equation of any asymptote and the coordinates of points where the curves cross the axes.

(4)

(*b*) Solve the equation $3\sinh 2x = 13 - 3e^{2x}$, giving your answer in the form $\frac{1}{2}\ln k$, where k is an integer.

(5)

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6. The plane P has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}$$

(*a*) Find a vector perpendicular to the plane P.

(2)

The line l passes through the point A (1, 3, 3) and meets P at (3, 1, 2).

The acute angle between the plane P and the line l is α.

(*b*) Find α to the nearest degree.

(4)

(*c*) Find the perpendicular distance from A to the plane P.

(4)

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7. The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} k & -1 & 1 \\ 1 & 0 & -1 \\ 3 & -2 & 1 \end{pmatrix}, \quad k \neq 1.$$

(*a*) Show that det $\mathbf{M} = 2 - 2k$.

(2)

(*b*) Find $\mathbf{M}^{-1}$, in terms of k.

(5)

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix

$$\begin{pmatrix} 2 & -1 & 1 \\ 1 & 0 & -1 \\ 3 & -2 & 1 \end{pmatrix}$$

The equation of l_2 is $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = 0$, where $\mathbf{a} = 4\mathbf{i} + \mathbf{j} + 7\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} + \mathbf{j} + 3\mathbf{k}$.

(*c*) Find a vector equation for the line l_1.

(5)

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8. The hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

(*a*) Use calculus to show that the equation of the tangent to H at the point $(a \cosh \theta, b \sinh \theta)$ may be written in the form

$$xb \cosh \theta - ya \sinh \theta = ab.$$

(4)

The line l_1 is the tangent to H at the point $(a \cosh \theta, b \sinh \theta)$, $\theta \neq 0$.

Given that l_1 meets the x-axis at the point P,

(*b*) find, in terms of a and θ, the coordinates of P.

(2)

The line l_2 is the tangent to H at the point $(a, 0)$.

Given that l_1 and l_2 meet at the point Q,

(*c*) find, in terms of a, b and θ, the coordinates of Q.

(2)

(*d*) Show that, as θ varies, the locus of the mid-point of PQ has equation

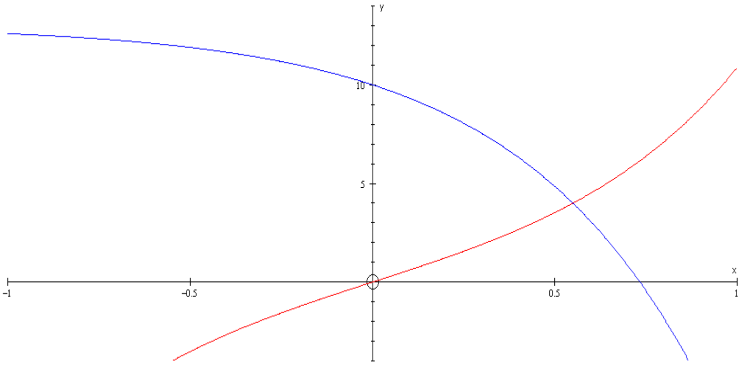
$$x(4y^2 + b^2) = ab^2.$$

(6)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	$\frac{dy}{dx}=6x^2$ and so surface area $=2\pi\int 2x^3\sqrt{(1+(6x^2)^2}dx$	B1
	$= 4\pi\left[\frac{2}{3\times36\times4}(1+36x^4)^{\frac{3}{2}}\right]$	M1 A1
	Use limits 2 and 0 to give $\frac{4\pi}{216}[13860.016-1]=806$ (to 3 sf)	DM1 A1
		5
2. (a) (i)	$\frac{dy}{dx}=\frac{x}{\sqrt{(1-x^2)}}+\arcsin x$	M1 A1 (2)
(ii)	At given value derivative $=\frac{1}{\sqrt{3}}+\frac{\pi}{6}=\frac{2\sqrt{3}+\pi}{6}$	B1 (1)
(b)	$\frac{dy}{dx}=\frac{6e^{2x}}{1+9e^{4x}}$	1M1 A1
	$=\frac{6}{e^{-2x}+9e^{2x}}$	2M1
	$=\frac{3}{\frac{5}{2}(e^{2x}+e^{-2x})+\frac{4}{2}(e^{2x}-e^{-2x})}$	3M1
	$\therefore \frac{dy}{dx}=\frac{3}{5\cosh 2x+4\sinh 2x}$ *	A1 cso
		(5) **8**
3. (a)	$x^2-10x+34=(x-5)^2+9$ so $\frac{1}{x^2-10x+34}=\frac{1}{(x-5)^2+9}=\frac{1}{u^2+9}$	B1
	(mark can be earned in either part (a) or (b))	
	$I=\int\frac{1}{u^2+9}du=\left[\frac{1}{3}\arctan\left(\frac{u}{3}\right)\right]$ | $I=\int\frac{1}{(x-5)^2+9}du=\left[\frac{1}{3}\arctan\left(\frac{x-5}{3}\right)\right]$	M1 A1
	Uses limits 3 and 0 to give $\frac{\pi}{12}$ | Uses limits 8 and 5 to give $\frac{\pi}{12}$	DM1 A1
		(5)
(b)	$I=\ln\left(\left(\frac{x-5}{3}\right)+\sqrt{\left(\frac{x-5}{3}\right)^2+1}\right)$ or $I=\ln\left(\frac{x-5+\sqrt{(x-5)^2+9}}{3}\right)$	M1 A1
	or $I=\ln\left((x-5)+\sqrt{(x-5)^2+9}\right)$	
	Uses limits 5 and 8 to give $\ln(1+\sqrt{2})$.	DM1 A1
		(4) **9**
4.		

Question Number	Scheme	Marks
(a)	$I_n = \left[\frac{x^3}{3}(\ln x)^n\right] - \int \frac{x^3}{3} \times \frac{n(\ln x)^{n-1}}{x} dx$ $= \left[\frac{x^3}{3}(\ln x)^n\right]_1^e - \int_1^e \frac{nx^2(\ln x)^{n-1}}{3} dx$ $\therefore I_n = \frac{e^3}{3} - \frac{n}{3} I_{n-1}$ *	M1 A1 DM1 A1cso (4)
(b)	$I_0 = \int_1^e x^2 dx = \left[\frac{x^3}{3}\right]_1^e = \frac{e^3}{3} - \frac{1}{3}$ or $I_1 = \frac{e^3}{3} - \frac{1}{3}\left(\frac{e^3}{3} - \frac{1}{3}\right) = \frac{2e^3}{9} + \frac{1}{9}$ $I_1 = \frac{e^3}{3} - \frac{1}{3} I_0$, $I_2 = \frac{e^3}{3} - \frac{2}{3} I_1$ and $I_3 = \frac{e^3}{3} - \frac{3}{3} I_2$ so $I_3 = \frac{4e^3}{27} + \frac{2}{27}$	M1 A1 M1 A1 (4) **8**
5. **(a)**	 Graph of $y = 3\sinh 2x$ Shape of $-e^{2x}$ graph Asymptote: $y = 13$ Value 10 on y axis and value 0.7 or $\frac{1}{2}\ln\left(\frac{13}{3}\right)$ on x axis	B1 B1 B1 B1 (4)
(b)	Use definition $\frac{3}{2}(e^{2x} - e^{-2x}) = 13 - 3e^{2x} \rightarrow 9e^{4x} - 26e^{2x} - 3 = 0$ to form quadratic $\therefore e^{2x} = -\frac{1}{9}$ or 3 $\therefore x = \frac{1}{2}\ln(3)$	M1 A1 DM1 A1 B1 (5) **9**

Question Number	Scheme	Marks
6. **(a)**	$\mathbf{n} = (2\mathbf{j}-\mathbf{k})\times(3\mathbf{i}+2\mathbf{j}+2\mathbf{k}) = 6\mathbf{i}-3\mathbf{j}-6\mathbf{k}$ o.a.e. (e.g. $2\mathbf{i}-\mathbf{j}-2\mathbf{k}$)	M1 A1 (2)
(b)	Line l has direction $2\mathbf{i}-2\mathbf{j}-\mathbf{k}$ Angle between line l and normal is given by $(\cos\beta \text{ or } \sin\alpha) = \frac{4+2+2}{\sqrt{9}\sqrt{9}} = \frac{8}{9}$ $\alpha = 90-\beta$ = 63 degrees to nearest degree.	B1 M1 A1ft A1 awrt (4)
(c)	Plane P has equation $\mathbf{r}.(2\mathbf{i}-\mathbf{j}-2\mathbf{k}) = 1$ Perpendicular distance is $\frac{1-(-7)}{\sqrt{9}} = \frac{8}{3}$	M1 A1 M1 A1 (4) **10**
7. **(a)**	$\text{Det}\,\mathbf{M} = k(0-2)+1(1+3)+1(-2-0) = -2k+4-2 = 2-2k$	M1 A1 (2)
(b)	$\mathbf{M}^T = \begin{pmatrix} k & 1 & 3 \\ -1 & 0 & -2 \\ 1 & -1 & 1 \end{pmatrix}$ so cofactors = $\begin{pmatrix} -2 & -1 & 1 \\ -4 & k-3 & k+1 \\ -2 & 2k-3 & 1 \end{pmatrix}$ (-1 A mark for each term wrong) $\mathbf{M}^{-1} = \frac{1}{2-2k}\begin{pmatrix} -2 & -1 & 1 \\ -4 & k-3 & k+1 \\ -2 & 2k-3 & 1 \end{pmatrix}$	M1 M1 A3 (5)
(c)	Let (x, y, z) be on l_1. Equation of l_2 can be written as $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 7 \end{pmatrix} + \lambda\begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$. Use $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$ with k = 2. i.e. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-2}\begin{pmatrix} -2 & -1 & 1 \\ -4 & -1 & 3 \\ -2 & 1 & 1 \end{pmatrix}\begin{pmatrix} 4+4\lambda \\ 1+\lambda \\ 7+3\lambda \end{pmatrix}$ $\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3\lambda+1 \\ 4\lambda-2 \\ 2\lambda \end{pmatrix}$ and so $(\mathbf{r}-\mathbf{a})\times\mathbf{b} = \mathbf{0}$ where **a** = **i** - 2**j** and **b** = 3**i** +4 **j** + 2**k** or equivalent **or** $\mathbf{r} = \mathbf{a}+\lambda\mathbf{b}$ where **a** = **i** - 2**j** and **b** = 3**i** +4 **j** + 2**k** or equivalent	B1 M1 M1 A1 B1ft (5) **12**

Question Number	Scheme	Marks
8. (a)	Uses $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b\cosh\theta}{a\sinh\theta}$ **or** $\frac{2x}{a^2} - \frac{2yy'}{b^2} = 0 \rightarrow y' = \frac{xb^2}{ya^2} = \frac{b\cosh\theta}{a\sinh\theta}$	M1 A1
	So $y - b\sinh\theta = \frac{b\cosh\theta}{a\sinh\theta}(x - a\cosh\theta)$	M1
	$\therefore ab(\cosh^2\theta - \sinh^2\theta) = xb\cosh\theta - ya\sinh\theta$ and as $(\cosh^2\theta - \sinh^2\theta) = 1$	A1cso
	$xb\cosh\theta - ya\sinh\theta = ab$ *	(4)
(b)	P is the point $(\frac{a}{\cosh\theta}, 0)$	M1 A1 (2)
(c)	l_2 has equation $x = a$ and meets l_1 at Q $(a, \frac{b(\cosh\theta - 1)}{\sinh\theta})$	M1 A1 (2)
(d)	The mid point of PQ is given by $X = \frac{a(\cosh\theta + 1)}{2\cosh\theta}$, $Y = \frac{b(\cosh\theta - 1)}{2\sinh\theta}$	1M1 A1ft
	$4Y^2 + b^2 = b^2\left(\frac{\cosh^2\theta + 1 - 2\cosh\theta + \sinh^2\theta}{\sinh^2\theta}\right)$	2M1
	$= b^2\left(\frac{2\cosh^2\theta - 2\cosh\theta}{\sinh^2\theta}\right)$	3M1
	$X(4Y^2 + b^2) = ab^2\left(\frac{(\cosh\theta + 1)(\cosh\theta - 1)2\cosh\theta}{2\cosh\theta\sinh^2\theta}\right)$	4M1
	Simplify fraction by using $\cosh^2\theta - \sinh^2\theta = 1$ to give $x(4y^2 + b^2) = ab^2$ *	A1cso
		(6) **14**